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Harmonic Analysis of Unsteady Transonic Flow

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Introduction

THE time averaged mean flow past an airfoil oscillating harmonically at transonic speeds differs from the steady flow obtained at rest. This difference depends on oscillation frequency and amplitude and arises because both mean and disturbance flowfields interact nonlinearly. For small oscillation amplitudes a time linearized description usually suffices: the mean flow is conveniently solved without reference to the unsteady flow and the harmonic flow is solved without explicit reference to the unsteady disturbance amplitude. For larger amplitudes, however, the wave back-interaction induced by the primary disturbance on the mean flow cannot be discounted because the modified mean flow in turn affects the evolution of the harmonic flowfield. This effect may be significant because transonic flows are inherently nonlinear. Linear theory,[1-4] which generally assumes small unsteady disturbance amplitudes in comparison to the overall thickness, expands the unsteady solution about the known transonic flow corresponding to the stationary airfoil. For oscillatory motions the mean flowfield is therefore defined by a simplified solution rendered independent of disturbance frequency and amplitude: the model implicitly assumes high-frequency oscillations where the mean flow and mean shock position effectively freeze in space. However, for low-frequency motions where large shock excursions are anticipated, the mean flow couples more strongly with the unsteady flow and linear theory may not apply. Thus the practical need arises for a rational harmonic formulation adaptable to unsteady transonic flutter and aeroelastic analyses yielding unstable amplitude as well as frequency boundaries. In this Note, a nonlinear harmonic approach to general unsteady oscillations in transonic flow is developed for those engineering applications where explicit amplitude and frequency information is required. The extent to which nonlinear feedback is significant is addressed, in particular, for the symmetric NACA 64A006 airfoil, unpitched, with a quarter chord oscillating flap executing large amplitude deflections in a subsonic freestream mildly to strongly supercritical. Details of the numerical algorithm are also outlined.

Analytical and Numerical Approach

The extent to which both mean and harmonic flowfields interact nonlinearly is measured for flapping motions by a nondimensional parameter ϵ formed on dividing the maximum angular deflection by the airfoil thickness ratio. Linear theory is obtained by expanding the unsteady velocity potential in powers of ϵ assuming a leading order nonlinear solution governed by the steady transonic small disturbance equation. Here, to retain the effect of nonlinear feedback within the framework of harmonic analysis, we expand the unsteady potential, the shock displacement, and the airfoil oscillation using real series where all complex Fourier wave components are accompanied by their conjugates, and, define a sequence of problems by equating coefficients of like harmonics. The resulting equations describe the harmonic interaction expected on physical grounds and provide an exact inviscid unsteady flow model not restricted to small amplitude oscillations. For simplicity, all harmonics higher than the primary are neglected. The resulting time transformed problem for the unsteady flow, then, reduces to the formulation of Refs. 3 and 4 except that all variable coefficients related to the mean flow are now formally unknown. The mean flow, in turn, satisfies the usual steady transonic flow equation modified by an added driving term whose strength is proportional to the product of ϵ^2 (which need not be small) and the streamwise derivative of an 0(1) unsteady pressure magnitude squared. The complete formulation for low-frequency flows appears in Ref. 5. Just how important the back interaction is will depend on the particular airfoil section, the value of ϵ, and the freestream Mach number M.

The coupled mean and unsteady flowfields can be solved using type-dependent relaxation methods. For simplicity our far-field boundary conditions assumed a zero unsteady potential everywhere excepting downstream infinity where we instead imposed ambient pressures; also, the mean (and not the unsteady) flowfield was solved conservatively. With the disturbance flow initialized to zero, the computational box for the mean potential was swept once assuming zero unsteady flow. The unsteady flow equations were solved next, using variable coefficients based on latest available values of mean potential, sweeping the box once; unsteady solutions so generated were then used to evaluate the driving term in the mean flow equation, which was solved next, and so on. The convergence speed of the present scheme appears to be slowed by the component of flow out of phase with the surface oscillation. On the other hand, the numerical stability is generally limited to low supercritical Mach numbers, typically $M < 0.85$, slow oscillations with reduced frequencies $k < 0.5$ based on semichord, and values of ϵ unity or less. Nevertheless, with enough trial and error, relaxation, and empirical mesh size optimization, a limited number of flows showing strong nonlinear coupling have been computed. These results are described next.

Calculated Results

The results obtained here correspond to the unsteady transonic flow past a symmetric NACA 64A006 airfoil at zero angle of attack with an oscillating quarter chord trailing-edge flap executing a maximum deflection angle δ. All calculations used the same grid system consisting of 80 streamwise and 60 transverse constant meshes with 20 taken over the chord. Overall dimensions of the computational box were approximately 5 × 5 chords. Tangency conditions were enforced along a chord embedded between two horizontal meshlines while Kutta's condition was applied, in the particular skeleton code used, slightly downstream of the trailing edge (hence, the nonzero but insignificant unsteady pressure jumps calculated at the actual trailing-edge position). This coarse mesh obviously requires refinement in high gradient regions but this was not pursued; one obvious consequence, for example, is an unrealistic smoothing of the hinge point pressure singularity. Three Mach numbers, 0.82, 0.85, and 0.90, and seven deflection angles, $\delta = 1, 2, \ldots, 7$ deg, were considered, and $k = 0.064$ throughout. Computations were terminated after 300 "cycles" where each cycle comprises of one mean flow sweep plus one unsteady flow sweep; at this stage, both mean and unsteady surface pressure solutions changed less than 5% per 50 cycles.

At Mach 0.82 stable results were obtained for $\delta = 1$-5 deg, while at Mach 0.85 and 0.90, the calculations converged only for $\delta = 1$ and 2 deg. For convenience we introduce the usual pressure coefficient

$$C_p = C_{p_0} + \epsilon (C^r_{p_1} \cos kt - C^i_{p_1} \sin kt)$$

where t is a normalized time, C_{p_0} is the steady pressure coefficient, $C^r_{p_1}$ is in phase with the boundary motion and $C^i_{p_1}$

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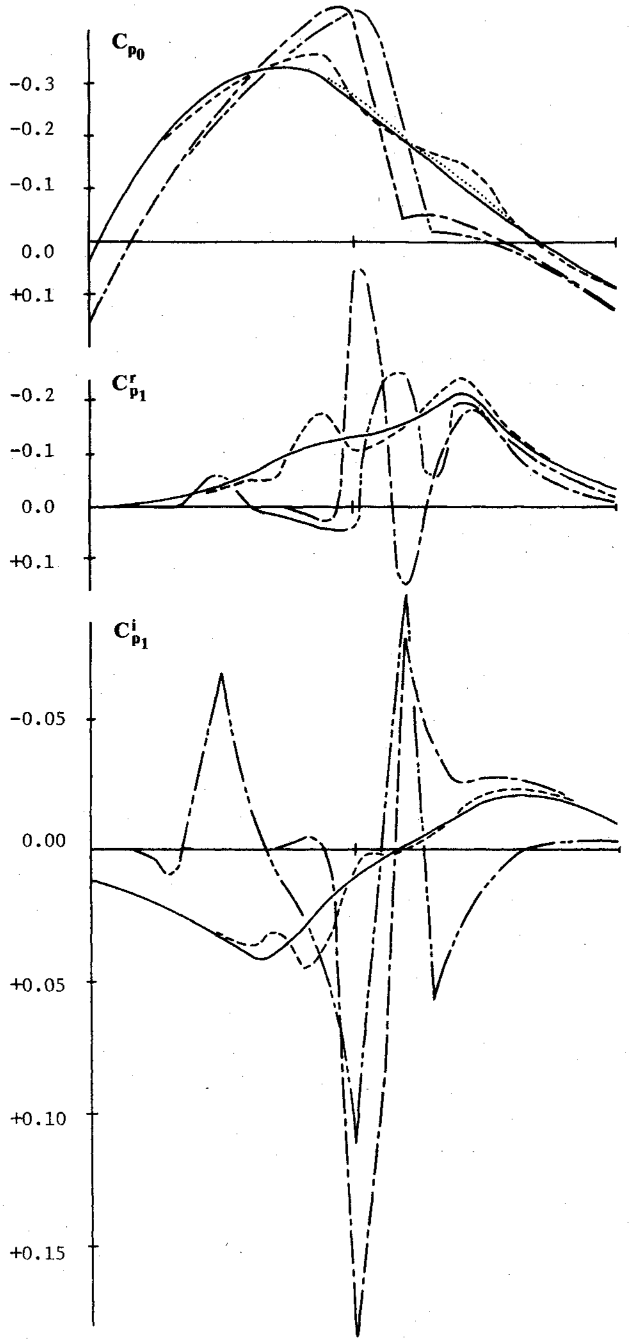

Fig. 1 Computed results for mean and unsteady surface pressure coefficients:———$M=0.82$, $\delta=1$ deg;·····$M=0.82$, $\delta=3$ deg;- - - $M=0.82$, $\delta=5$ deg;— - — -$M=0.90$, $\delta=1$ deg; and— - - — - - $M=0.90$, $\delta=2$ deg. Unsteady results at $M = 0.82$ for $\delta = 1$ and 3 deg are indistinguishable.

is out of phase. Figure 1 concisely presents results obtained for Mach 0.82 and 0.90. In the former case the effect of nonlinear feedback is unimportant even at $\delta = 3$ deg, which corresponds to $\epsilon = 0.87$; at $\delta = 5$ deg, where $\epsilon = 1.45$, the back interaction is clearly evident. Results for Mach 0.85, not shown here, indicate that the back interaction is unimportant even at $\delta = 2$ deg, agreeing with results documented in earlier work[6] (Ref. 6 furnishes a more complete discussion of the numerical procedure). However, at Mach 0.90 the effects of mean and harmonic flow coupling are extremely pronounced. With increasing δ, the mean shock location moves downstream toward the hinge point, forming a rearward region of high spatial flow gradients. The plateau region seen in the $C^r_{p_1}$ Mach 0.82 curve for $\delta = 1$ deg, characteristic of linear theory[3,4] as well as experimental small amplitude results,[7] with increasing δ and M develop into highly contrasting crests and troughs. With increasing δ, holding M fixed, the flap oscillations influence more of the upstream flow, possibly because the increased unsteady disturbance energy enables the wave to travel farther upstream around the mean supersonic bubble. As noted, our unsteady solutions contained rapidly varying peaks and valleys. For the coarse mesh used, the integrity of these solutions was monitored by observing their (stable) development with iteration number, by checking the smoothness of the C_{p_0} curve, and by comparing, with δ fixed, qualitative changes in the unsteady pressure obtained as a function of Mach number and iteration history. Experimental results for large deflection angles, unfortunately, are not available for comparison; however, for small angles, our computed results here and in Ref. 6 for $C^r_{p_1}$ and $C^i_{p_1}$ agree qualitatively with the results of Tijdeman.[7]

Discussion and Closing Remarks

For the NACA 64A006 flapping airfoil considered here, our results for $M = 0.82$, $\delta = 1, 2,$ and 3 deg, and $M = 0.85$, $\delta = 1$ and 2 deg indicate that the nonlinear coupling between mean and oscillatory flowfields is insignificant. For these weakly supercritical flows linear theory still applies despite values of ϵ near unity. Hence one potential application of nonlinear harmonic analysis: it can be used to assess the extent to which simpler linear approaches apply for large oscillations (this is a definite asset in aeroelastic and flutter analyses). Our results also show how the anticipated strong dependence of $C^r_{p_1}$ and $C^i_{p_1}$ on ϵ for large ϵ and/or M is easily obtained; more detailed analysis on a finer mesh should, however, be pursued.

The linearity of the unsteady response obtained in our Mach 0.85 calculations was somewhat unexpected. For $\delta = 1$ deg our results showed a definite shock wave near midchord with C_{p_0} increasing 0.22 over three meshes, approximately, suggesting that the nonlinear coupling considered here should become even more important for larger values of δ; for $\delta = 2$ deg, calculations indicated a downstream shock movement less than 3% of chord and no noticeable change in shock strength or computed unsteady chordwise loading. Similar conclusions are noted in Ref. 6 for $M = 0.80$, $k = 0.064$ and $M = 0.85$, $k = 0.24$, for deflections up to 3 deg. Note that the GTRAN2 time marching scheme discussed in Ref. 8 can also be used to study mean and harmonic flow interactions except that end results must be then Fourier analyzed (this approximate factorization method applies to general oscillation amplitudes and frequencies). In light of the foregoing observations, we present a result not noted in Ref. 8. GTRAN2 calculations for a NACA 64A010 section unpitched at Mach 0.82 with a 1 deg flap deflection showed shock excursions oscillating sinusoidally about the same mean position for very high to very low reduced frequencies. One might have anticipated different mean positions for different frequencies, in particular, at the lower frequencies; but, perhaps, this is not so surprising because here $\epsilon = 0.17$ only. The linearity of the unsteady response for small values of ϵ is also evident from Tijdeman's experiments[7] and is discussed at length in a recent review article.[9] Finally, note that the same mathematical ideas applied to inviscid parallel flow stability, show that the back-interaction effect smooths the inflectional mean profile and leads to stable equilibrium solutions for all waves that are "self-excited" on a linear basis.[10]

Acknowledgments

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Behavior of the Turbulent Energy Equation at a Fixed Boundary

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I. Introduction

THE behavior of the turbulent energy equation at a fixed boundary has an important significance to the modeling of turbulence phenomena. Coantic[1] expressed the turbulent pressure and velocity components as a Taylor series with respect to distance from a wall. He concluded that the dissipation at the wall was approximately 0.095, when normalized by friction velocity (U_τ) and kinematic viscosity (ν). Townsend[2] corrected Laufer's[3] results to make dissipation balance gradient diffusion, with the other terms of the energy equation zero. He cited a dimensional argument to justify his correction. Hanjalić and Launder[4] used a similar power series to estimate dissipation near the wall.

The work reported in this Note is an extension of the previous work of Coantic[1] and Townsend,[2] and it is based on an internal axisymmetric incompressible turbulent flow. The object of this Note is to collect the scattered work of this type into a single source.

II. Analysis

A standard cylindrical coordinate system is used with variation of mean quantities being restricted to the radial direction. Upper case letters represent mean quantities, with lower case representing turbulent quantities; the symbols u, v, w represent the axial (x), radial (r), and transverse (ϕ) velocity components, respectively.

A power series expansion, with respect to distance from the wall ($y=R-r$), is used to describe the turbulence field. Subject to the continuity equation and the no-slip boundary conditions, the following expansions are used:

$$u=a_1(x,\phi,t)y+\ldots \tag{1}$$

$$v=b_2(x,\phi,t)y^2+\ldots \tag{2}$$

$$w=c_1(x,\phi,t)y+\ldots \tag{3}$$

$$p=d_0(x,\phi,t)+d_1(x,\phi,t)Y+\ldots \tag{4}$$

Eckelmann[5] reported fluctuations in the molecular shear stress, for directions parallel to the wall, at the wall. Hence u and w must contain a linear term.

The turbulent energy equation, in cylindrical coordinates, is

$$\begin{aligned}
&\overbrace{\frac{\nu}{U_T{}^4}\left[\overline{u^2}\frac{\partial U}{\partial x}+\overline{uv}\left(\frac{\partial U}{\partial x}+\frac{\partial U}{\partial r}\right)+\overline{v^2}\frac{\partial V}{\partial r}+V\frac{\overline{w^2}}{r}\right]}^{\text{Production}}\\
&+\frac{\nu}{U_T{}^4}\overbrace{\left[\frac{\partial}{\partial x}(U\overline{q^2})+\frac{1}{r}\frac{\partial}{\partial r}(rV\overline{q^2})\right]}^{\text{Advection}}\\
&+\frac{\nu}{U_T{}^4}\overbrace{\left[\frac{\partial}{\partial x}\overline{u(q^2+p)}+\frac{1}{r}\frac{\partial}{\partial r}\overline{rv(q^2+p)}\right]}^{\text{Diffusion}}\\
&-\frac{\nu^2}{U_T{}^4}\overbrace{\left[\frac{\partial^2}{\partial x^2}(\overline{q^2})+\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial}{\partial r}\overline{q^2}\right)\right]}^{\text{Gradient Diffusion}}\\
&+\frac{\nu^2}{U_\tau^4}\overbrace{\left[\left(\overline{\frac{\partial u}{\partial x}}\right)^2+\left(\overline{\frac{1}{r}\frac{\partial u}{\partial \phi}}\right)^2+\left(\overline{\frac{\partial u}{\partial r}}\right)^2\right.}^{\text{Dissipation}}\\
&\quad+\left(\overline{\frac{\partial v}{\partial x}}\right)^2+\left(\overline{\frac{\partial v}{\partial r}}\right)^2+\left(\overline{\frac{1}{r}\frac{\partial v}{\partial \phi}}\right)^2\\
&\quad+\left(\overline{\frac{\partial w}{\partial x}}\right)^2+\left(\overline{\frac{\partial w}{\partial r}}\right)^2+\left(\overline{\frac{1}{r}\frac{\partial w}{\partial \phi}}\right)^2\\
&\quad\left.-\frac{4}{r}\left(\overline{\frac{w}{r}\frac{\partial v}{\partial \phi}}+\frac{\overline{v^2+w^2}}{r}\right)\right]=0
\end{aligned} \tag{5}$$

where $q^2=(u^2+y^2+w^2)/2$ and the over bar indicates a long time average.

When the power series is substituted into the energy equation and evaluated at the wall ($y=0$) the following results.

Production

$$\Big|_{y=0}=0 \tag{6}$$

Advection

$$\Big|_{y=0}=0 \tag{7}$$

Diffusion

$$\Big|_{y=0}=0 \tag{8}$$

Gradient Diffusion

$$\Big|_{y=0}=\frac{\nu^2}{U_T{}^4}\left(\overline{a_1^2(x,\phi,t)}+\overline{c_1^2(x,\phi,t)}\right) \tag{9}$$

Dissipation

$$\Big|_{y=0}=\frac{\nu^2}{U_T{}^4}\left(\overline{a_1^2(x,\phi,t)}+\overline{c_1^2(x,\phi,t)}\right) \tag{10}$$

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